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A Note on Yield Curve Steepness and Solvency II Capital Requirement

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For the calculation of interest rate capital charges under Solvency II shifts of the risk-free interest rate curve defined by EIOPA need to be added to a bond's specific yield curve. By repricing the bond with the shifted yield curve the respective Solvency Capital Requirement (SCR) value can be obtained. One practical difficulty that arises in this context is that bonds' yield curves are not not observable and need to be empirically fitted. Our study *Solvency II Interest Rate Charges - Why Yield Curve Choice Matters* describes the effect of different yield curve models on resulting SCR.
This note provides some mathematical intuition for the following empirical observation in our study: when taking the steepness of bonds' yield curves into account we typically obtain lower SCR interest rate values than when modelling with a flat yield curve. The SCR is also lower for more advanced yield curve models such as Polynomial, Spline, Nelson-Siegel and Svensson than in the case where a yield curve is modelled by a parallelly shifted 'risk-free' curve. We suspect that the latter observation is related to the steepness of credit spreads that are more adequately captured by the advanced models.

Let us consider two ways of modelling a bond's price P_0, a model (A) and a model (B) with following specifications:

$$\begin{aligned} P_0^{(A)} &= \sum_t \frac{c_t}{(1+y)^t} \\ P_0^{(B)} &= \sum_t \frac{c_t}{(1+a+bt)^t} \end{aligned} \tag{1}$$

where both models provide the same price, $P_0 = P_0^{(A)} = P_0^{(B)}$. Here, c_t is the cash flow in year t, y is the bond's yield to maturity, and a and b are the intersect and slope of the linear model. For simplicity, we assume a coupon frequency of 1 and consider all parameters as being known (i.e. a and b are calibrated values).
Let us assume that interest rates are shifted by a constant s at all maturities t[1]. This yields

$$\begin{aligned} P_+^{(A)} &= \sum_t \frac{c_t}{(1+y+s)^t} \\ P_+^{(B)} &= \sum_t \frac{c_t}{(1+a+bt+s)^t} \end{aligned}$$

[1] This assumption is a good proxy in the current low interest rate environment as the minimum interest rate shift size of 1% defined in Article 166/2 of Commission Delegated Regulation (EU) 2015/35 is applicable to most tenors of the risk-free curve.

Before turning to more formal results, it is worthwhile noting that intuitive, one would expect that for $b > 0$ it should hold that $P_+^{(A)} < P_+^{(B)}$ which means a steeper yield curve should result in a smaller price decline and thus a lower SCR. Let us next turn to a more formal analysis.

Consider the case of a small positive b. Then we have $y = a + \delta y$ where $\delta y = \mathrm{O}(b)$,

$$P_0^{(A)} = \sum_t \frac{c_t}{(1+a)^t} - \delta y(1+a)^{-1} \sum_t \frac{tc_t}{(1+a)^t}$$
$$P_0^{(B)} = \sum_t \frac{c_t}{(1+a)^t} - b(1+a)^{-1} \sum_t \frac{t^2 c_t}{(1+a)^t}$$

This yields

$$y = a + b \left[\sum_t \frac{tc_t}{(1+a)^t} \right]^{-1} \sum_t \frac{t^2 c_t}{(1+a)^t} + \mathrm{O}(b^2)$$

$$\begin{aligned}
\frac{\partial y}{\partial a} &= 1 + b(-1) \left[\sum_t \frac{tc_t}{(1+a)^t} \right]^{-2} \left[\sum_t \frac{-t^2 c_t}{(1+a)^{t+1}} \right] \sum_t \frac{t^2 c_t}{(1+a)^t} \\
&\quad + b \left[\sum_t \frac{tc_t}{(1+a)^t} \right]^{-1} \sum_t \frac{-t^3 c_t}{(1+a)^t} \\
&= 1 + \frac{b}{1+a} \left[\sum_t \frac{tc_t}{(1+a)^t} \right]^{-2} \\
&\quad \times \left\{ \left[\sum_t \frac{t^2 c_t}{(1+a)^t} \right]^2 - \left[\sum_t \frac{tc_t}{(1+a)^t} \right] \left[\sum_t \frac{t^3 c_t}{(1+a)^t} \right] \right\}
\end{aligned}$$

Defining the weight

$$w_t = \frac{tc_t}{(1+a)^t} \,,$$

and

$$\langle f \rangle = \frac{\sum_t f_t w_t}{\sum_t w_t}$$

one obtains

$$\frac{\partial y}{\partial a} = 1 + \frac{b}{1+a} \left\{ \langle t \rangle^2 - \langle t^2 \rangle \right\} < 1 \qquad (2)$$

since $\langle t^2 \rangle - \langle t \rangle^2 = \mathrm{Var}(t) > 0$ (apart from the trivial case).
This means that for small positive b, increasing a by a small amount $\delta a = s$ one should increase y by a smaller amount, $\delta y < s$ to maintain the relation $P_0^{(A)} = P_0^{(B)}$. Increasing y by s which is larger than the value needed to maintain the equality one decreases the corresponding price, hence (at least in this special case) one has indeed $P_+^{(A)} < P_+^{(B)}$.
Moreover, we see from (2) that $\partial y / \partial a$ decreases with increasing b. Thus, SCR is higher for steeper yield curves (even for the same yield to maturity y).

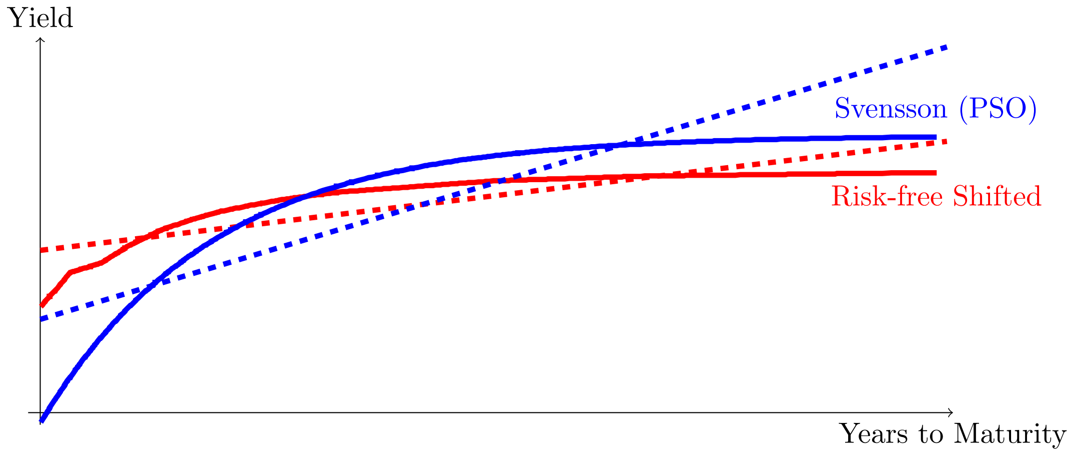

Figure 1: *When proxying yield curves by linear models, it can be shown that a higher steepness causes lower changes in bond prices when the linear yield curve is shifted.*

Conclusion

Our study *Solvency II Interest Rate Charges - Why Yield Curve Choice Matters* has shown empirically that Solvency II capital charges (Solvency Capital Requirement - SCR) are lower when bonds' underlying yield curves are steep. This note confirms this observation for linearised yield curve models and shows analytically that higher steepness indeed leads to lower SCRs.