Classifying Solvency Capital Requirement Contribution of Collective Investments under Solvency II

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Depending on the current risk exposure of an insurance company, the impact of buying an additional unit of a fund on an insurer's overall Solvency II capital charges, i.e., the Solvency Capital Requirement (SCR), will differ. We call this impact the fund's SCR contribution and show in which boundaries it lies if only the fund's aggregate sub-SCR figures are known but not the risk exposures of the insurance company buying the fund. The upper bound of this range, the worst case SCR contribution, can be used as a conservative measure to assess a fund's Solvency II risk contributions or to assign them to different Solvency II risk categories. We believe that providing funds' worst case SCR contributions can be useful information to insurance companies when screening from a broad investment universe.

1 Introduction

The EU-wide insurance regulation framework Solvency II (see European Union (2009)), in effect since January 1, 2016, sets out various reporting requirements to insurance companies. According to the Delegated Regulation (EU) 2015/35 in European Union (2015b), a look-through of collective investments is generally required, increasing the operational reporting complexity on the side of various market participants. In order to support insurance companies in their regulatory obligations, a standard Solvency II fund reporting template, commonly known as the tripartite template, was developed by three fund associations: the BVI in Germany, club AMPERE in France, and The Investment Association in the UK.

The tripartite template (tpt) used by the fund management industry to report Solvency II relevant figures is designed to be at the holdings level (for further details see The Investment Association (2016)). As the aggregation of the Solvency Capital Requirement (SCR) submodules is done on the insurers' balance sheets, no average SCR figures are required to be reported for funds. And there is a clear reason for this: the impact on an insurance company's overall SCR not only depends on the risk exposures of the fund, but also on how well they are diversified compared to the risks taken by the insurance company. In other words, one cannot tell the exact SCR contribution of a fund without knowing the risk exposures of the insurance company investing in it. Consequently, for each insurance company, a fund's SCR contribution will be different. However, it is still possible to calculate a fund's *worst-* and *best-case* SCR contribution without any information on the fund's investor. This paper provides an analytical solution for the former and numerical results for the latter measure.

Moreover, we show how to categorise funds into low, medium, and high Solvency II risk profiles and how to interpret these figures.

Related Literature The importance of the details of aggregation of SCR has been emphasized by Devineau and Loisel (2009) and Asimit et al. (2016).[1] We differ from this literature by focusing on lower bounds for SCR. In a wider sense, this article is also related to the literature providing support for the square-root formula used for the Solvency II SCR calculation (see, e.g., Christiansen et al. (2012)).
For an overview of the Solvency II regulations and a survey of the early literature, see Eling et al. (2007). See also Klein (2013) for a more recent overview of the literature on Solvency II and insurance regulations in general.

2 Specification

The five SCR submodules reported in the tpt report are the interest rate, spread, equity, currency risk, and property. They are calculated for each security individually and aggregated with all other positions held by the insurance company.

Consider an insurance company that has a vector of sub-SCRs, denoted by $\boldsymbol{w}$. The insurance company invests in a fund with a normalized sub-SCR vector denoted by $\boldsymbol{x}$. With normalization we describe the sub-SCR that corresponds to a EUR 1 investment in the fund. In the following, we will make the approximation that the sub-SCRs increase linearly with the weight of the assets, which is a reasonable approximation given that insurance companies typically have a low exposure to equity.[2] We will also assume that the sub-SCRs are all positive.[3] Denote by $\boldsymbol{C}$ the correlation matrix of the SCR Market Module according to the Solvency 2 regulations:

$$\boldsymbol{C} = \begin{bmatrix} 1 & A & A & A & 0 & 0.25 \\ A & 1 & 0.75 & 0.75 & 0 & 0.25 \\ A & 0.75 & 1 & 0.5 & 0 & 0.25 \\ A & 0.75 & 0.5 & 1 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 & 1 \end{bmatrix}. \tag{1}$$

The parameter A as defined in Article 164 (3) of the Delegated Regulation (EU) 2015/35 in European Union (2015b) equals 0.5 if the liabilities' interest rate sensitivity dominates (e.g., the liabilities' duration is longer than that of the assets), otherwise A equals 0. The rows (and

[1] Devineau and Loisel (2009) demonstrate that the standard formula for SCR can be considered as a first order approximation of the result of an internal model and propose a nested simulations model to converge the standard model and the internal model of risk. Asimit et al. (2016) consider proportional risk transfers in insurance groups.

[2] This approximation would be perfect if the investment in equity of both the insurance company and the fund had the same fraction of investment in type 1 and type 2 equities.

[3] Having no short positions is a sufficient, but not necessary, condition for all sub-SCRs to be positive: a company with, e.g., short positions in some equity still has a positive equity sub-SCR if the long positions dominate.

columns) stand for the following sub-SCRs: Interest rate, Equity, Property, Spread, Concentration, and Currency. Note that the concentration sub-SCR will be omitted in the following, as is also done in the standard tpt fund report. This is due to the fact that concentration sub-SCRs are not simply additive between holdings.
An insurance company investing α euros in the fund has the SCR

$$S(\boldsymbol{w} + \alpha\boldsymbol{x})$$

where the SCR function is

$$S(\boldsymbol{y}) = \sqrt{\boldsymbol{y}'\boldsymbol{C}\boldsymbol{y}}.$$

We want to rank the different funds according to how much an investment in the fund contributes to the insurance company's SCR on the margin. Formally, we are interested in

$$f(\boldsymbol{w};\boldsymbol{x}) \equiv \left.\frac{\partial S(\boldsymbol{w} + \alpha\boldsymbol{x})}{\partial\alpha}\right|_{\alpha=0},$$

which we will refer to as the fund's *SCR contribution*. The SCR contribution measures how much an investment of one additional Euro in that fund increases the insurance company's SCR. A desirable feature of the concept of the SCR contribution is that it is invariant to scaling the insurance company's portfolio proportionally, as

$$f(\lambda\boldsymbol{w};\boldsymbol{x}) = f(\boldsymbol{w};\boldsymbol{x}), \qquad \text{for } \lambda > 0, \tag{2}$$

i.e., it depends only on the direction of the vector $\boldsymbol{w}$ and not on its length.
While the SCR contribution does not vary with a rescaling of $\boldsymbol{w}$, it does depend on the composition of the insurance company's portfolio, that is, the direction of the vector $\boldsymbol{w}$. Nonetheless, we can provide worst case values of a fund's SCR contributions, that is, an upper bound of the SCR contribution of a fund that is independent of the insurance company's sub-SCR's $\boldsymbol{w}$.

3 Worst-Case SCR Contribution

The SCR contribution of a fund is highest if $\boldsymbol{x}$ and $\boldsymbol{w}$ are parallel vectors, i.e., if the investment structure of the fund and the insurance company are the same. Intuitively, this is because the fund does not provide additional diversification. In the worst-case, a fund's SCR contribution is $\sqrt{\boldsymbol{x}'\boldsymbol{C}\boldsymbol{x}}$.
We state this result for the worst-case (i.e., maximum) SCR contribution formally in the following proposition.

Proposition 1. For all $\boldsymbol{w}$,

$$\left.\frac{\partial S(\boldsymbol{w} + \alpha\boldsymbol{x})}{\partial\alpha}\right|_{\alpha=0} = \frac{\boldsymbol{x}'\boldsymbol{C}\boldsymbol{w}}{\sqrt{\boldsymbol{w}'\boldsymbol{C}\boldsymbol{w}}} \leq \sqrt{\boldsymbol{x}'\boldsymbol{C}\boldsymbol{x}}, \tag{3}$$

where the inequality becomes an equality if and only if $\boldsymbol{x}$ and $\boldsymbol{w}$ are parallel, that is, $\boldsymbol{x} = \beta\boldsymbol{w}$ for some scalar β.

Proof. Since $\boldsymbol{C}$ is positive definite[4], we can define an inner product space with the inner product

$$\langle\boldsymbol{a},\boldsymbol{b}\rangle = \boldsymbol{a}'\boldsymbol{C}\boldsymbol{b}.$$

By the Cauchy–Schwarz inequality,

$$|\langle\boldsymbol{x},\boldsymbol{w}\rangle|^2 \leq \langle\boldsymbol{x},\boldsymbol{x}\rangle\langle\boldsymbol{w},\boldsymbol{w}\rangle,$$

[4] The matrix $\boldsymbol{C}$ is positive definite for both values of A, with the eigenvalues $[1.0, 0.5, 2.4706, 1.1822, 0.6877, 0.1595]$ for $A = 0$ and $[1.0, 0.5, 2.8950, 0.8733, 0.5761, 0.1555]$ for $A = 0.5$.

for any two vectors $\boldsymbol{x}$ and $\boldsymbol{w}$, where equality holds if and only if $\boldsymbol{x}$ and $\boldsymbol{w}$ are parallel. Taking square roots and observing that taking the absolute value is redundant here since the matrix elements C_{ij} and the vector components x_i and w_j are non-negative, we get

$$\boldsymbol{x}'\boldsymbol{C}\boldsymbol{w} \leq \sqrt{\boldsymbol{x}'\boldsymbol{C}\boldsymbol{x}}\sqrt{\boldsymbol{w}'\boldsymbol{C}\boldsymbol{w}}. \tag{4}$$

This implies the statement in the proposition. □

4 Best-Case SCR Contribution—Numerical Results

In the following, we pick examples of values for $\boldsymbol{x}$ and numerically maximize the SCR contribution with $\boldsymbol{w}$ as the maximization variable.[5] This serves as a confirmation of our analytical results. More interestingly, we also numerically *minimize* the SCR contribution. This gives us the best-case (i.e., minimal) SCR contribution.
In the following, we define three funds with the following sub-SCRs:

Table 1: Simulating SCR contributions for funds with different risk profiles

Fund's Risk Profile	SCR submodule: Interest Rate	Spread	Equity	Property	Currency Risk	numerical simulation $A = 0$: min	max	$A = 0.5$: min	max
High	1%	8%	23%	0%	0%	1.0%	29.5%	8.0%	30.0%
Medium	3%	9%	0%	0%	0%	3.0%	9.5%	3.0%	10.8%
Low	2%	3%	0%	0%	0%	1.3%	3.6%	1.3%	4.4%

with A being defined in Article 164 (3) of the Delegated Regulation.
Table 1 reports three funds with a high, medium and low risk profile as well as the corresponding sub-SCRs' values. The numerical results show that the maximum values are equal to the analytical solution $\sqrt{\boldsymbol{x}'\boldsymbol{C}\boldsymbol{x}}$.[6] This also confirms that the maximum values for $A = 0.5$ are higher, which is a consequence of the positive values given in $\boldsymbol{C}$ and $\boldsymbol{x}$. Moreover, the table shows that the minimum SCR contribution can be significantly lower and inconsistent with the fund's maximum SCR contribution: the minimum SCR for $A = 0$ is higher for the medium risk portfolio (3.0%) than for the other two (1.0% and 1.3%, respectively).
The numerical results in Table 1 show the minimum SCR contribution a fund can make to the overall SCR of an insurance company. For the above example of three funds with 'High', 'Medium', and 'Low' risk profiles, one can numerically identify the insurance company's risk exposures $\boldsymbol{w}$ where the SCR contribution is minimal. This is shown in Table 2 for the cases $A = 0$ and $A = 0.5$.
The worst case, the upper limit in Eq. (4), is reached when $\boldsymbol{w}$ and $\boldsymbol{x}$ are parallel. The situation is different for the lower limit, assuming that the components of $\boldsymbol{x}$ and $\boldsymbol{w}$ are non-negative, since in this case an anti-parallel $\boldsymbol{x}$ and $\boldsymbol{w}$ are not allowed. Performing a numerical minimization in the concrete cases discussed below, in all cases considered we have found that the minimum appears at one of the corners of the allowed regions, where only one component of $\boldsymbol{w}$ is

[5] Since the SCR contribution is invariant to a rescaling of $\boldsymbol{w}$, we keep the length of the $\boldsymbol{w}$ vector fixed for the maximization.
[6] By plugging in the SCR submodule contributions from the table into Equation (3) and using the values from (1) for $\boldsymbol{C}$ (leaving out the Concentration contribution, i.e., the fifth row and the fifth column), we get for the maximum SCR contributions 29.5%, 9.5%, and 3.6%, respectively, for $A = 0$. For $A = 0.5$, we get 30.0%, 10.8%, and 4.4%. These values are equal to the values in the last but two and the last columns in the table.

Table 2: Insurance company sub-SCRs where fund SCR contribution is minimal.

	SCR submodule					
	Int. Rate	Spread	Equity	Property	Currency Risk	SCR Contribution
$\boldsymbol{w}(A=0)$	1%	0%	0%	0%	0%	1.0%
$\boldsymbol{x}(High)$	1%	8%	23%	0%	0%	
$\boldsymbol{w}(A=0.5)$	0%	0%	0%	0%	1%	8.0%
$\boldsymbol{w}(A=0)$	1%	0%	0%	0%	0%	3.0%
$\boldsymbol{x}(Medium)$	3%	9%	0%	0%	0%	
$\boldsymbol{w}(A=0.5)$	0%	0%	0%	0%	1%	3.0%
$\boldsymbol{w}(A=0)$	0%	0%	0%	0%	1%	1.3%
$\boldsymbol{x}(Low)$	1%	8%	23%	0%	0%	
$\boldsymbol{w}(A=0.5)$	0%	0%	0%	0%	1%	1.3%

different from zero. This suggests that it would be enough to consider the five corners, $\boldsymbol{w} = (1,0,0,0,0),\ldots,(0,0,0,0,1)$ to find the minimum. However, one can show that this claim is not true in general.

Using the scaling property (2) one can reformulate the problem into finding the minimum of $\Phi(\boldsymbol{w}) = (\boldsymbol{x}'C\boldsymbol{w})$ on the surface of the ellipsoid $\boldsymbol{w}'C\boldsymbol{w} = 1$. By an orthogonal transformation, one can transform C to diagonal form: $C = ODO'$. Then by rescaling the components of the resulting vector $O\boldsymbol{w}$, one transforms the ellipsoid to a sphere of radius 1. The reformulated problem simplifies to finding the minimum of the function $\Psi(\boldsymbol{v}) = \boldsymbol{y}'\boldsymbol{v} = v_1$ on the surface of the unit sphere $||\boldsymbol{v}|| = 1$. Here we took $y = (1,0,\ldots,0)$ without loss of generality. The vector $\boldsymbol{v}$ is related by the linear transformations described above to $\boldsymbol{w}$ and C. By these transformations the boundaries $w_i = 0$ of the constraints are mapped into hyper-planes in the space $(v_1, v_2, \ldots)$. Consider for simplicity the 3-dimensional case. In this case the intersection of the unit sphere with the three planes corresponding to $w_i = 0$ results in a spherical triangle. The problem is reduced to finding the lowest point within this spherical triangle when the highest point, the "north pole" is within the triangle.

In the first step we make the orthogonal transformation

$$C = ODO'\,, \quad D = \text{diag}[d_1, d_2, \ldots] \tag{5}$$

and

$$\boldsymbol{w} = O\boldsymbol{z} \tag{6}$$

With this

$$w'Cw = \boldsymbol{z}'D\boldsymbol{z} = \boldsymbol{v}'\boldsymbol{v} \tag{7}$$

where

$$\boldsymbol{v} = D^{1/2}\boldsymbol{z}\,, \tag{8}$$

If the diagonal matrix D is proportional to the unit matrix then the angles of the spherical matrix are $90°$. Since the maximum within the triangle, the lowest point is necessarily one of the corners. However, the stretching in (8) in general changes the angles, and one can finally have a spherical triangle with angles larger than $90°$. In this case, the lowest point of the triangle can be on one of its sides.

Hence, without further assumptions, one cannot rule out the latter case, and one has to perform the numerical minimization instead of just calculating the value of $f(\boldsymbol{w};\boldsymbol{x})$ at the corner points.

5 A Fund's Solvency II Risk Categorization

A simple categorization of a fund's SCR contribution could be done into low, medium, and high. This requires the definition of SCR contribution thresholds λ_1 and λ_2 where $\lambda_1 < \lambda_2$. We define a fund's Solvency II risk profile as

$$\text{Solvency II risk profile} = \begin{cases} \text{low}, & \text{if } SCR_{maxcontr} \leq \lambda_1 \\ \text{medium}, & \text{if } \lambda_1 < SCR_{maxcontr} \leq \lambda_2 \\ \text{high}, & \text{if } \lambda_2 < SCR_{maxcontr} \end{cases}$$

with

$$SCR_{maxcontr} = \sqrt{\boldsymbol{x}'\boldsymbol{C}_{A=0.5}\boldsymbol{x}}\,.$$

As we are looking for a conservative calculation of the maximum SCR contribution, and since

$$\boldsymbol{x}'\boldsymbol{C}_{A=0.5}\boldsymbol{x} \geq \boldsymbol{x}'\boldsymbol{C}_{A=0}\boldsymbol{x}$$

for $\boldsymbol{x} \geq 0$, we can omit the $A = 0$ case defined in Article 164f.[7] Note also that the risk categorization assumes a maximally conservative scenario, where $\boldsymbol{x}$ is parallel to $\boldsymbol{w}$, i.e., the fund does not 'improve' the diversification of the existing balance sheet. The specification of the thresholds λ_1 and λ_2 is of course arbitrary. We suggest taking round numbers for simplicity, e.g., $\lambda_1 = 10\%$ and $\lambda_2 = 30\%$.

6 Practical Considerations

In practice, Solvency II tripartite reports are generated on shareclass level to reflect market risk adequately to investors. As typically funds' shareclasses differ in currency hedging and fee structure, it can be a pragmatic approach for fully hedged shareclasses to aggregate sub-SCR figures on fund level excluding Currency sub-SCRs. A further reason to exclude aggregated Currency sub-SCR from a report is due to the fact that investors' home currency may differ (i.e. may not be EUR). The remaining sub-SCR figures in the tripartite template to be aggregated are Interest Rates, Spread, Equity, and Property. If the shareclasses are fully hedged in home currency, the Currency sub-SCR will be 0%; if the shareclass is fully hedged into foreign currency, the Currency sub-SCR typically will be 25%.[8]
Moreover, there are cases in which the interpretation of the Solvency II standard model rules is ambiguous when applied to complex instrument types and strategies in investment funds. To illustrate this by an example let us consider a 'curve trade' where a 5 years short future on a AA rated bond with a market value of 6 million EUR is opposed to a 30 years long future on the same bond with market value of 1 million EUR. While the same underlying basis risk allows for a netting of the two positions, there is still an ambiguity in the process. If one first nets durations, the aggregate trade will have a duration of roughly 0 years. However, if one were to calculate Spread sub-SCRs for both trades separately and then net the values, the resulting Spread sub-SCR would be around -150'000 EUR. There are of course numerous other examples to illustrate the limitations of the Solvency II insurance regulation when applied to asset management products. Typically, a conservative way to avoid questions of interpretation is by classifying such funds as Type 2 Equity and omitting a look-through reporting of the fund.

[7] That $\boldsymbol{x}'\boldsymbol{C}_{A=0.5}\boldsymbol{x} \geq \boldsymbol{x}'\boldsymbol{C}_{A=0}\boldsymbol{x}$ holds can be seen from the fact that $\sum_{ij} x_i C_{ij} x_j$ is weakly increasing in C_{ij} for all i and j if $\boldsymbol{x} \geq 0$.

[8] See European Union (2015a) for the details of the exceptions.

7 Conclusions

This paper shows that aggregating sub-SCR numbers for funds does make sense. With a conservative interpretation, where a fund investor's balance sheet sub-SCRs have the same ratios as the fund's sub-SCRs (worst-case scenario), the aggregation can be done according to the correlation matrix defined in Article 164 of the Delegated Regulation using the parameter $A = 0.5$.

Finally, aggregating SCRs on the fund level allows for classifying funds into, e.g., low, medium, and high risk profiles. Reporting aggregate fund figures may therefore not only be operationally simpler as no Non-Disclosure Agreements may be needed for such figures, but could also be regarded as useful information to insurance companies in the fund selection process.

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