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The Effect of Transactions on Capital Charges under the Swiss Solvency Test

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The standard Market model of the Swiss Solvency Test defines an insurance company's target capital as a function of its exposure to risk factors on the asset and liability side of the balance sheet. By buying and selling assets an insurer's asset manager can actively change the target capital and therefore the solvency coverage ratio. Obviously, commiting a transaction will depend on several factors such as return and risk expectations, liquidity, asset class and issuer constraints, etc., however, the impact on the solvency coverage ratio should also be assessed. This paper derives a transaction's effect on the overall solvency coverage ratio for the linear FINMA model and incorporates liquidity constraints for non-tradable assets. The model is based on the official FINMA document in [1] and applies the notation in [2].

The Model

An insurer's target capital depends on the distribution of its risk bearing capital ("Risikotragendes Kapital", RTK hereafter) which is given by

$$\mathrm{RTK}_{t+1} = \mathrm{RTK}_t(z_t + X_{t+1})\,,$$

where z_t is a vector of economic variables at time t defined by FINMA and X_{t+1} is its change so that $z_{t+1} = z_t + X_{t+1}$. Note that a 12 months period is considered between t and $t+1$ and X_{t+1} is modelled as a multivariate normal distributed vector

$$X \sim \mathcal{N}(\mu_X, \Sigma_X)\,.$$

As described in [2], the change of risk bearing capital over time depends on X, the assets' first and second order sensitivities towards X given by δ and Γ, the assets' price change over time $p_{t+1}(z_{t+1}) - p_t(z_t)$, any cash flow CF_i within 12 months, the probabililty weighted effect of historical financial crisis scenarios and on the units u_i held in an asset i:

$$\begin{aligned}\Delta\mathrm{RTK}_{t+1} = \sum_i & u_i \times \{p_{t+1,i}(z_t) - p_{t,i}(z_t) + \mathrm{CF}_i \\ & + \delta'_{t+1,i}X + \frac{1}{2}X'\Gamma_{t+1,i}X \\ & + \sum_n (p_{t+1,i}(z_t + X_n) - p_{t,i}(z_t)) \times Y_n\}\,.\end{aligned} \tag{1}$$

Due to the quadratic term given by Γ and the financial market scenario term in the expression in (1) is not normally distributed.[1] In order to obtain analytical results for a transaction's impact the target capital we simplify this equation to its linear form in the following.
The linear version of (1) can be written as

$$\begin{aligned}\Delta\mathrm{RTK}_{t+1} &= \sum_i u_i \times \{p_{t+1,i}(z_t) - p_{t,i}(z_t) + \mathrm{CF}_i \\ &\quad + \delta'_{t+1,i}X\} \\ &= \sum_i u_i \times \left[\Delta p_{t+1,i}(z_t) + \delta'_{t+1,i}X\right]\,,\end{aligned} \tag{2}$$

where $\Delta p_{t+1,i}$ corresponds to the price's time effect (e.g. pull-to-par effect) $p_{t+1,i}(z_t) - p_{t,i}(z_t)$ and cash flow CF_i of asset i within the next 12 months. Note that the linearized version of the model follows a univariate normal distribution given by

$$\Delta\mathrm{RTK}_{t+1} \sim \mathcal{N}(\mu_{\Delta\mathrm{RTK}}, \sigma_{\Delta\mathrm{RTK}})$$

[1] For details on the notation refer to [2].

with parameters

$$\mu_{\Delta\mathrm{RTK}} = \sum_i u_i \left(\Delta p_{t+1,i}(z_t)\right) + \delta' \mu_X \,, \qquad (3)$$

$$\sigma_{\Delta\mathrm{RTK}} = \sqrt{\delta' \Sigma_X \delta} \qquad (4)$$

where

$$\delta = \sum_i u_i \delta_i \,.$$

The target capital TC, which is the minimal amount that has to be covered by an insurance company's risk bearing capital, is given by the ΔRTK distribution's expected shortfall on the 1% confidence level. For a normally distributed ΔRTK this corresponds to

$$f(u) \equiv \mathrm{TC} = 2.665 \times \sigma_{\Delta\mathrm{RTK}} - \mu_{\Delta\mathrm{RTK}} \,. \qquad (5)$$

The Effect of Transactions on Target Capital

In the following we examine a transaction from asset 1 (reduce) into asset 2 (increase) and call this a switch. A switch of equal CHF value corresponds to

$$\Delta u_1 p_1 + \Delta u_2 p_2 = 0 \,, \qquad (6)$$

where in the above equation p_i stands for $p_{t,i}(z_t)$. The sign of switching from asset 1 into asset 2 means $\Delta u_1 < 0$ (and $\Delta u_2 > 0$, assuming positive prices p_1, p_2).
The change of target capital by this switch is given by

$$\begin{aligned} df &= \frac{\partial f}{\partial u_1}\Delta u_1 + \frac{\partial f}{\partial u_2}\Delta u_2 \\ &= \left(\frac{1}{p_1}\frac{\partial f}{\partial u_1} - \frac{1}{p_2}\frac{\partial f}{\partial u_2}\right) p_1 \Delta u_1 \end{aligned} \qquad (7)$$

and the partial derivative of f is

$$\begin{aligned} \frac{\partial f}{\partial u_i} =& 2.665 \times \left(\delta' \Sigma_X \delta\right)^{-1/2} \delta_i' \Sigma_X \delta \\ & - \Delta p_{t+1,i}(z_t) - \delta_i' \mu_x \end{aligned} \qquad (8)$$

The sign of the expression within brackets in (7) is relevant for assessing the impact of this switch on the target capital. For

$$\left(\frac{1}{p_1}\frac{\partial f}{\partial u_1} - \frac{1}{p_2}\frac{\partial f}{\partial u_2}\right) > 0$$

the target capital decreases by the proposed switch from 1 to 2. The maximal decrease from such a switch is obtained when 1 is chosen as the asset with maximal $p_i^{-1}\partial f/\partial u_i$ while 2 as the asset with the corresponding minimal value.
Allowing for a simultaneous reallocation of assets under the condition $\sum_i u_i p_i = \mathrm{const}$ the direction of optimal change (the steepest descent) is given by

$$\Delta u_i \propto \frac{\partial f}{\partial u_i} - \lambda p_i \qquad (9)$$

for all assets i. The Lagrange multiplier λ is given by

$$\lambda = \frac{1}{\sum_j p_j^2} \sum_j p_j \frac{\partial f}{\partial u_j} \qquad (10)$$

The optimal direction to decrease target capital is given in Eqs. (9) together with (10).

Constraints on Non-tradable Assets

Some assets on the balance sheet may be illiquid or there are other reasons for an asset manager to exclude them from the transaction analysis. It is for example quite expensive in terms of transaction costs to switch from equities into direct real estate investments and therefore adding a constraint of constant units on such investments makes sense for practical applications.
Adding the constraint of $\Delta u_n = 0$ on a non-tradable asset n can be easily implemented in Eqs. (9) and (10) by simply excluding the n-th dimension. The remaining directions obtained by (9) will correspond to simultaneous switches ($\sum_i \Delta u_i p_i = 0$) while assuming that units held in asset n do not change. This can be generalized to any set of non-tradable assets by excluding the respective dimensions from Eqs. (9) and (10).

Basel III Capital Charges

In addition to the standard market model, capital charges according to the Basel III framework are applied on insurance companies' assets (see [3]). These charges depend on the asset type (e.g. the asset class and further characteristics of an asset) and rating as described in Appendix 2 in [4]. Denoting the (relative) capital charge on asset i by c_i, Eq. (5) has to be adapted to

$$f(u) = 2.665 \times \sigma_{\Delta\mathrm{RTK}} - \mu_{\Delta\mathrm{RTK}} + \sum_i u_i p_{t,i} c_i \qquad (11)$$

and the change of target capital in Eq. (8) rewritten to:

$$\begin{aligned} \frac{\partial f}{\partial u_i} =& 2.665 \times \left(\delta' \Sigma_X \delta\right)^{-1/2} \delta_i' \Sigma_X \delta \\ & - \Delta p_{t+1,i}(z_t) - \delta_i' \mu_x + p_{t,i} c_i \end{aligned} \qquad (12)$$

Conclusion

This paper has derived some basic equations of the linear SST market model described in [1]. They can be used for fast and extensive analyses such as the examination of marginal SST contributions of a broad set of assets.
Our empirical tests with company data – not covered in this paper – show that the linear model's marginal SST contributions provide similar results (same sign and order of magnitude) as values of the exact calculation of the model. As for some applications (e.g. a quick filtering of a large investment universe) an approximate calculation is often sufficient, we believe to provide a useful input to practitioners by this paper and a documentation of our tools.

References

[1] FINMA (2013): Wegleitung zum SST-Marktrisiko-Standardmodell

[2] B. Kovacs, A. Niedermayer, D. Niedermayer (2014): Implementing the SST Standard Market Model, SolvencyAnalytics Working Paper Series 2014-04 (01)

[3] FINMA (2013): Wegleitung zum SST-Kreditrisiko-Standardmodell

[4] Verordnung über die Eigenmittel und Risikoverteilung für Banken und Effektenhändler (2013), Anhang 2